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EXECUTION-ERROR MODELING AND ANALYSIS OF THE CASSINI-HUYGENS SPACECRAFT THROUGH 2007

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The Cassini-Huygens spacecraft arrived at Saturn in 2004, beginning a four-year tour. Much of mission's continued success at Saturn can be attributed to the excellent performance of the propulsion systems and attitude control. In order to better understand this performance, the Cassini Navigation Team has continued to analyze and refine the execution-error models for the propulsion systems. This paper documents the evolution of the execution-error models employed for maneuvers, along with the analysis, procedures, and software associated with the model development.

INTRODUCTION

After a seven-year interplanetary cruise, the Cassini-Huygens spacecraft entered a Saturnian orbit in July 2004 to study the planet and its many moons. Now separated from the Huygens probe, the Cassini orbiter is nearing the end of a planned four-year tour of the Saturnian system and about to begin an extended mission of two years; it continues to perform remarkably well. Much of the success of the spacecraft can be attributed to the excellent performance of the propulsion systems and attitude control. The spacecraft accomplishes maneuvers through the use of two independent propulsion systems; the bi-propellant Main Engine Assembly (MEA) for performing large burns, and the Reaction Control System (RCS) thrusters for small trajectory corrections called Orbit Trim Maneuvers (OTM). In order to better understand this performance, the Cassini Navigation Team continues to analyze and refine the execution-error models of both the MEA and RCS systems. These updates help maneuver predictions during operations and help improve analysis of maneuvers in future tour segments. This paper documents the evolution of the execution-error models that have been employed for maneuvers, along with the analysis, procedures, and software associated with the development of these models. The latest model, designated 2007-02, is also presented, along with the maneuver data that was considered, a description of the modeling process, and plots illustrating how the maneuver errors relate to the model. The maneuver data set spans from 1998, during the interplanetary cruise, through September 2007, three years into the Saturn prime tour.

HISTORY OF MODELS

Throughout the lifespan of this mission there have been several MEA and RCS execution-error models, as seen in Tables 1 and 2. They continue to be in the form of the Gates model,[1] which accounts for magnitude and pointing errors. In 2000, the MEA component of the 1996 (pre-launch) model[2] was updated via an analysis of seven MEA maneuvers performed during the first two years

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of the interplanetary cruise. The resulting 2000 model[3] was utilized for maneuvers from April 2000 during cruise to February 2006 into the Saturn tour.

In March 2006, the model was again revisited by considering most maneuvers executed up through November 2005 with OTM-044 (40 MEA and 13 RCS).[4] This model, designated 2006-01, was used from March 2006, starting with OTM-053, to August 2007, ending with OTM-124. Besides reducing the MEA magnitude and proportional error parameters and the RCS magnitude error parameters, this model introduced a few changes in spacecraft operations. The computed RCS proportional-magnitude bias was removed by reducing the predicted thrust level by 1.5% starting with the design of OTM-047.[5] Also, beginning with OTM-069, the MEA proportional-magnitude bias seen in the 2006-01 study was removed by increasing the MEA accelerometer scale factor by 0.06%.[6]

The 2007-01 model has been in use since September 2007 with OTM-125. Based on the 2006-01 model, it reduced the RCS fixed-pointing error standard deviation from 3.5 mm/s to 0 mm/s as RCS OTM turns use the Reaction Wheel Assembly (RWA), which do not produce a turn $\Delta\mathbf{V}$. The 2007-01 model was meant to be an interim model until the next execution-error analysis.[7]

A recent analysis of maneuvers from interplanetary cruise through September 2007 with OTM-129 has culminated in the design of the 2007-02 model. At the time of this paper, this model is under review and may replace the 2007-01 model. The development process of the 2007-02 model will be discussed in detail throughout this paper.

Table 1 History of MEA Execution-Error Models (1-σ)

		1996 Model	2000 Model	2006-01 Model	2007-01 Model	2007-02 Model
Magnitude	Proportional (%)	0.35	0.2	0.04	0.04	0.02
	Fixed (mm/s)	10.0	10.0	6.5	6.5	5.0
Pointing (per axis)	Proportional (mrad)	10.0	3.5	1.0	1.0	0.6
	Fixed (mm/s)	17.5	17.5	4.5	4.5	3.0

Table 2 History of RCS Execution-Error Models (1-σ)

		1996 Model	2000 Model	2006-01 Model	2007-01 Model	2007-02 Model
Magnitude	Proportional (%)	2.0	2.0	0.7	0.7	1.2
	Fixed (mm/s)	3.5	3.5	0.9	0.9	0.8
Pointing (per axis)	Proportional (mrad)	12.0	12.0	12.0	12.0	5.5
	Fixed (mm/s)	3.5	3.5	3.5	0	0

MODEL DESCRIPTION

Maneuver execution errors for Cassini-Huygens are in the form of the Gates model.[1] By applying a maximum-likelihood estimator, the Gates-model parameters for magnitude and pointing errors can be computed, along with the biases associated with the errors. These two components are tied together; the execution-error model assumes that some or all of the biases have been removed.

Gates Model

The Gates model accounts for four independent error sources, fixed- and proportional-magnitude errors and fixed- and proportional-pointing errors. The direction of pointing errors is assumed to have a uniform distribution across 360°. Each of the four sources is assumed to have a Gaussian distribution, so each parameter represents the standard deviation for that error source and each error source is assumed to have a zero mean.

Maximum-Likelihood Estimator

The Gates-model parameters are determined herein with maximum-likelihood estimation.[8] In a coordinate system whose x axis is parallel to the desired $\Delta\mathbf{V}$, the Gates model gives the following covariance

$$P_{gates} = \begin{pmatrix} \sigma_1^2 + y^2\sigma_2^2 & 0 & 0 \\ 0 & \sigma_3^2 + y^2\sigma_4^2 & 0 \\ 0 & 0 & \sigma_3^2 + y^2\sigma_4^2 \end{pmatrix} \tag{1}$$

where y is the magnitude of the maneuver $\Delta\mathbf{V}$, σ_1 and σ_2 are the fixed and proportional Gates-model parameters for magnitude, and σ_3 and σ_4 are the fixed and proportional Gates-model parameters for pointing. For any given maneuver, the Gates model is Guassian $N(0, P_{gates})$, but for a set of maneuvers with different $\Delta\mathbf{V}$ magnitudes, it is not Guassian because the standard deviation is a function of y. As a result, the standard deviation of the execution-error model is not simply the standard deviation of the samples; it must be obtained using a method like maximum-likelihood estimation. The procedure for this method is to derive a likelihood expression as a function of the model parameters and then maximize the likelihood of the given observations.

First, the probability density function (pdf) for the magnitude error is

$$f_m(x, y, \sigma_1, \sigma_2) = \left[2\pi(\sigma_1^2 + y^2\sigma_2^2)\right]^{-1/2} \exp\left[-\frac{1}{2}\frac{(x-\mu_m)^2}{\sigma_1^2 + y^2\sigma_2^2}\right] \tag{2}$$

where x is the magnitude error, μ_m is the mean magnitude error, and exp is the exponential function. Then, the likelihood function for magnitude errors, L_m, is defined as the product of evaluations of f_m for each measurement:

$$L_m(\sigma_1, \sigma_2) = \prod_{i=1}^{N} f_m(x_i, y_i, \sigma_1, \sigma_2) \tag{3}$$

Likewise, for the pointing error, a two-dimensional vector, the pdf is

$$f_p(x, y, \sigma_3, \sigma_4) = \left[\sqrt{2}\pi(\sigma_3^2 + y^2\sigma_4^2)\right]^{-1} \exp\left[-\frac{1}{2}\frac{(x-\mu_p)^2}{\sigma_3^2 + y^2\sigma_4^2}\right] \tag{4}$$

where x is the length of the pointing error vector in units of speed, and μ_p is the mean pointing error. The likelihood function for pointing errors, L_p, is then defined as follows:

$$L_p(\sigma_3, \sigma_4) = \prod_{i=1}^{N} f_p(x_i, y_i, \sigma_3, \sigma_4) \tag{5}$$

A weighted maximum-likelihood approach is constructed by raising each term in the likelihood function to a power. For the magnitude errors, the exponent is the inverse of the 1-σ uncertainty. For pointing errors, the uncertainty is two-dimensional, so the inverse of the standard deviation of the error along the pointing-error direction is used. The Gates-model parameters for magnitude errors are found by maximizing L_m; likewise for pointing errors L_p. It is often easier to maximize the natural logarithms of L_m and L_p, rather than the likelihood functions directly (adding numbers instead of multiplying numbers). Since the natural logarithm function is a monotonically increasing function, the solutions will be the same.[8]

Based on the form of Eqs. 2 and 4, only two measurements are required to determine the parameters (solving two unknowns requires requires two equations). It follows then that with more measurements, more accurate estimates will be produced.

PROCESSING OF MANEUVER DATA

In assembling the maneuver execution-error data that will be fitted, it may seem appropriate to just simply subtract the reconstructed $\Delta\mathbf{V}$ from the design $\Delta\mathbf{V}$ in an inertial coordinate system like EME2000 to obtain the maneuver execution error. However, this approach does not provide insight into the source of the error and may not be consistent with the orbit determination (OD).

One issue is there are events associated with each maneuver that, although they may not be part of the maneuver $\Delta\mathbf{V}$ design, cannot be cleanly separated out in the OD process. Consequently, the $\Delta\mathbf{V}$ for each maneuver includes the design $\Delta\mathbf{V}$ (burn and turns) plus any $\Delta\mathbf{V}$ events related to the maneuver, including but not limited to the following:

- Pointing-bias-fix turns[9] (for MEA burns)
- Deadband tightening and limit cycling (for RCS burns)
- Deadbanding of the spacecraft
- Reaction Wheel Assembly (RWA) / RCS transitions (for MEA burns)
- RWA biases (rotation rate changes) within a few hours of the burn (usually for RCS burns)

This $\Delta\mathbf{V}$ (design $\Delta\mathbf{V}$ plus associated $\Delta\mathbf{V}$ events) will be herein referred to as the expected $\Delta\mathbf{V}$.

A second issue is the choice of coordinate system for expressing the errors. Since each maneuver $\Delta\mathbf{V}$ is in a different inertial direction yet is controlled by the onboard cut-off algorithm and attitude control system, spacecraft body-fixed coordinates are a natural choice for analyzing the execution errors. A spacecraft coordinate frame already exists for Cassini, as seen in Figure 1: $X_{S/C}$, $Y_{S/C}$, and $Z_{S/C}$. The $Z_{S/C}$ axis points from the high gain antenna to the MEA, the $X_{S/C}$ axis points away from where Huygens was attached, and the $Y_{S/C}$ axis completes the right-handed system. However, a coordinate system with an axis parallel to the expected $\Delta\mathbf{V}$ is preferred. The compromise is the thrust-vector-control (TVC) coordinate frame with Z_{TVC} parallel to the expected $\Delta\mathbf{V}$, X_{TVC} parallel to the projection of $X_{S/C}$ onto the plane perpendicular to Z_{TVC}, and Y_{TVC} completing the right-handed system. The plane perpendicular to Z_{TVC} is referred to herein as the pointing plane.[9] With this type of coordinate frame, the execution error can be expressed with two perpendicular components, magnitude and pointing. Magnitude errors are computed simply by differencing the lengths of the reconstructed and expected $\Delta\mathbf{V}$ vectors. Pointing errors are the vector differences of the reconstructed and expected $\Delta\mathbf{V}$s projected onto the pointing plane. They are given in X_{TVC} and Y_{TVC} components in m/s as they represent $\Delta\mathbf{V}$ errors. Use of angular units is reserved for the proportional component of the pointing errors.

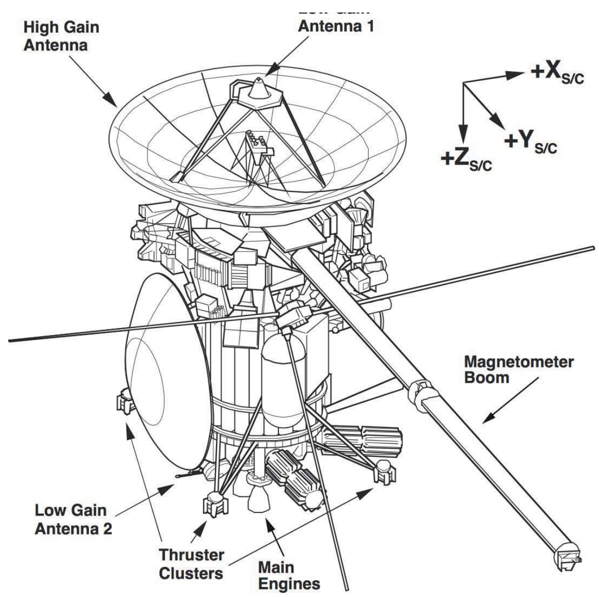


Figure 1 Cassini-Huygens Spacecraft

With the development of the 2007-02 model, several new considerations were made in the processing of the maneuver data. These included adjusting the expected MEA burn $\Delta\mathbf{V}$s with a corrected accelerometer scale factor and the expected RCS burn $\Delta\mathbf{V}$s with better thrust predictions.

The MEA uses an onboard accelerometer to compute the commanded $\Delta\mathbf{V}$. The accelerometer scales its data with the scale factor, producing a acceleration measurement. Those measurements are accumulated to provide increments of $\Delta\mathbf{V}$; the burn is terminated when the commanded $\Delta\mathbf{V}$ is achieved. The accelerometer scale, therefore, affects the executed $\Delta\mathbf{V}$. If it is too large, the executed $\Delta\mathbf{V}$ will be too small, and vice-versa. The ratio of the estimated accelerometer scale factor to the onboard value can be used to correct the expected $\Delta\mathbf{V}$ of MEA burns:

$$\Delta V_{corr.\ expected} = \Delta V_{expected} + (c_{MEA} - 1)\Delta V_{BURN} \tag{6}$$

where c_{MEA} is the accelerometer scale factor correction ratio ($c_{MEA} = \frac{Estimated\ Acc.\ S.\ F.}{Onboard\ Acc.\ S.\ F.}$). This ratio will be equal to 1 for maneuvers that were executed using the latest estimate of the accelerometer scale factor. In the 2007-02 study, MEA maneuvers prior to OTM-069 had be corrected.

Unlike MEA, $\Delta\mathbf{V}$s for RCS maneuvers are computed via a virtual accelerometer, which measures increments of time, not $\Delta\mathbf{V}$. Increments of burn time are converted to increments of $\Delta\mathbf{V}$ via the classic rocket equation, which is where the onboard thrust primarily influences the algorithm. When the accumulation of these increments reaches the desired $\Delta\mathbf{V}$, the burn is cut off. Hence, if the onboard thrust value is too large, than the executed $\Delta\mathbf{V}$ will be too small, and vice-versa. Discrepancies between the onboard and predicted thrust values have usually been due to either onboard values not being updated since the previous maneuver or onboard values being updated with earlier predicts. Operationally, these differences have been eliminated starting with OTM-100, when ground software started automatically providing spacecraft commands to update the thrust with the latest predicted value. In order to correct the expected $\Delta\mathbf{V}$ maneuver, the predicted thrust should be accounted for. Analogous to c_{MEA}, this is accomplished by computing the ratio of the predicted thrust value to the onboard thrust value and applying it to the RCS burn $\Delta\mathbf{V}$:

$$\Delta V_{corr.\ expected} = \Delta V_{expected} + (c_{RCS} - 1)\Delta V_{BURN} \tag{7}$$

where c_{RCS} is the thrust correction ratio ($c_{RCS} = \frac{Predicted\ Thrust}{Onboard\ Thrust}$). For the 2007-02 analysis, most of the RCS burns prior to OTM-100 had to be adjusted. Using this ratio to correct the $\Delta\mathbf{V}$ assumes linearity of $\Delta\mathbf{V}$ with thrust.

EXECUTION-ERROR ANALYSIS SOFTWARE

To process effectively the increasing amount of maneuver data during the Saturn tour for execution-error analysis, a software tool was developed by T. Goodson and S. Wagner of the Cassini-Huygens Navigation team. This tool was first used to design the 2006-01 model, which involved the processing of maneuvers performed through November 2005 (40 MEA and 13 RCS maneuvers up to OTM-044), a large undertaking at the time. Fast forwarding to September 2007, the number of maneuvers has more than doubled (71 MEA and 37 RCS maneuvers up to OTM-129). The Execution-Error Analysis Tool has now become an essential part of this modeling process.

The Execution-Error Analysis Tool, herein referred to as ExeTool, reads design and reconstructed maneuver data to produce separate MEA and RCS execution-error models that fit the data, along with tabulated data of the maneuver magnitude and pointing errors and plots showing how the errors fit/disagree with the generated models. It is comprised mainly of Perl and Matlab scripts and is largely based on the software architecture established with the Maneuver Automation Software (MAS) and the maneuver analysis tools developed within the maneuver team.[10]

All data generated for a maneuver design and reconstruction needed to be easily accessible to the program. This required the collaboration of other scripts outside of the ExeTool to provide the necessary inputs to the program, by creating database files for each maneuver. The database files are a collection of inputs that 'define' the maneuver characteristics, which are either entered directly by the user or are outputs from other programs. A 'maneuver status' database file provides a user-defined list of the maneuvers (which also supplies the maneuver order) and whether they were MEA, RCS, or cancelled. The generated databases include the following for each performed maneuver:

- Maneuver design files that define the $\Delta\mathbf{V}$s included in the expected $\Delta\mathbf{V}$.
- OD reconstruction files that provide the estimated maneuver $\Delta\mathbf{V}$s and associated $\Delta\mathbf{V}$ events, and corresponding covariances.
- Files that list the parameters pointing to each maneuver's burn and turn $\Delta\mathbf{V}$s and other associated $\Delta\mathbf{V}$ events, specifically for constructing the expected $\Delta\mathbf{V}$s and estimated $\Delta\mathbf{V}$s and covariances

These files have been somewhat automated with the execution of related software, such as the MAS software for files related to the expected $\Delta\mathbf{V}$s, and other maneuver analysis software for files related to the estimated $\Delta\mathbf{V}$s and covariances. Manual edits are usually reserved for adding or removing $\Delta\mathbf{V}$ events or giving correction values to the expected $\Delta\mathbf{V}$s (see previous section "Processing of Maneuver Data").

A configuration file is the main mechanism for user inputs to the program, once the above files have been provided. Inputs to ExeTool include the following:

- Range of maneuvers to include in model generation.
- List of maneuvers to exclude from the data set.
- List of the $\Delta\mathbf{V}$ events to exclude in constructing the $\Delta\mathbf{V}$s and covariances.
- Locations of the maneuver design, OD reconstruction, and database files.

- Options for weighting the maneuvers by the estimated $\Delta\mathbf{V}$ standard deviations.
- Options for removing magnitude/pointing biases in the generated model.
- General table and plotting options.

Outputs from ExeTool include logs that list the $\Delta\mathbf{V}$ events included in the construction of the expected $\Delta\mathbf{V}$s and estimated $\Delta\mathbf{V}$s and covariances, tables that list the magnitude and pointing errors and associated uncertainties for each maneuver (see Tables 3-5), tables that contain the execution-error models for MEA and RCS (see Table 6), tables that contain the computed biases to magnitude and pointing for MEA and RCS (see Tables 7- 8), and the following types of plots are generated for both MEA and RCS:

- Magnitude error vs. maneuver magnitude (see Figures 3 and 5).
- TVC Pointing plane with pointing-error ellipses (see Figures 4 and 6).
- Pointing-error magnitude vs. maneuver magnitude.
- Pointing-error directions.
- Magnitude-error cumulative distribution functions (CDFs) (see Figure 7).

Figure 2 shows how the various input sources to ExeTool are used and the processing of the maneuver data to produce the execution-error model and bias estimations. This setup has allowed quick parametric studies and the automation of larger studies, such as a study of how the exclusion of a single maneuver's data can affect the generated models and biases.

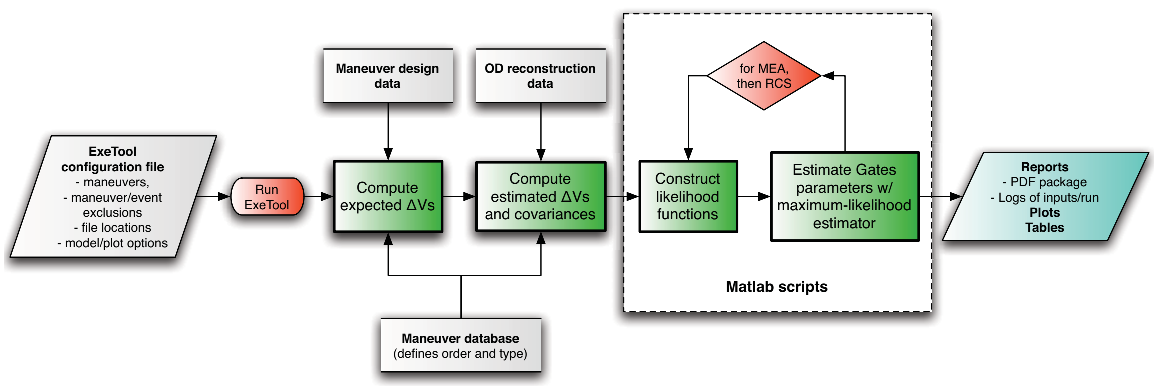


Figure 2 ExeTool Flow Diagram

MANEUVER EXECUTION DATA THROUGH 2007

Most maneuvers performed during interplanetary cruise through September 2007 were used in the determination of the 2007-02 maneuver execution-error model: specifically, 71 MEA and 37 RCS maneuvers have been performed. Tables 3-5 list the magnitude and pointing errors of the MEA and RCS maneuvers used in this execution-error study. Maneuvers that are listed but excluded from the modeling process are italicized. The $\Delta\mathbf{V}$ magnitude for each maneuver is the expected $\Delta\mathbf{V}$; it includes the design $\Delta\mathbf{V}$ (burn and turns) plus all $\Delta\mathbf{V}$ events related to the maneuver (e.g., deadband tightening, Reaction Wheel Assembly (RWA) / RCS transitions, etc.). Also, the 1-σ pointing uncertainty numbers are 1-σ ellipse dimensions (semi-major axis $\times$ semi-minor axis) with orientation angles (relative to the pointing plane X_{TVC} axis). Most maneuvers that are currently excluded are still being investigated to determine if they simply need a correction to a predicted value or if they are simply results of unusual circumstances. Only a couple of maneuvers are permanently excluded from the analysis. TCM-01 is excluded because, as the first maneuver, there are already several aspects which would need complicated corrections, such as an error in the algorithm for on-board estimation of the maneuver magnitude.[11] TCM-02 and TCM-07 are excluded while some old information about onboard parameter settings is being recovered. SOI is excluded from the data set since it is difficult to interpret SOI's execution errors into magnitude and pointing. The spacecraft was commanded to rotate about 45° over the course of the burn.[12]

Finally, as the mission has progressed, some changes in operations affect this analysis. An example is a change in the turn rate for maneuvers. To save hydrazine, the turn rate for yaw turns is being decreased for MEA maneuvers since OTM-111. RCS maneuvers use RWA for turns and don't consume hydrazine. However, if the yaw turn angle is larger than 120°, then a slightly higher rate is used. It isn't currently know how compatible these two post-OTM-111 rates are with the existing execution-error model nor how they should be handled in future analyses of execution errors.

2007-02 EXECUTION-ERROR STUDY

The 2007-02 execution-error model was based on the analysis of 106 executed maneuvers (71 MEA and 37 RCS), from the beginning of the interplanetary cruise to Saturn into the fourth year of Saturn tour with OTM-129. Using maneuvers performed from December 1998 through September 2007, a less conservative execution-error model was developed (see Refs. 4 and 5). This model, designated 2007-02, at the time of this paper is under review and will possibly replace the 2007-01 model being used in operations.

Several maneuvers have been tentatively excluded from the maximum-likelihood estimation. As in any parameter estimation process, some data appear to be outliers and are inconsistent with the vast majority of the data set. In this analysis, the overall data set is fairly small and these outliers beg further investigation so that they may be included in the analysis with confidence. OTMs 44 and 103 gave undue influence to the magnitude bias terms. OTMs 53, 81-BU, 89, and 106 influenced the pointing-error bias terms in contradicting directions. All of these were excluded, but are listed in Tables 3-5.

Maneuver Error Plots

Figures 3 and 5 show magnitude error as a function of maneuver magnitude for the MEA and RCS maneuvers, respectively, used in the 2007-02 study. The error bars show the 1-σ uncertainties in the

Table 3 MEA Maneuver Execution Errors (December 1998 - September 2005)

Maneuver *(Excluded)*	Maneuver Epoch (UTC/SCET)	Expected $\Delta\mathbf{V}^*$ (m/s)	Magnitude		Pointing		
			Mag. Error (mm/s)	1-σ Uncert. (mm/s)	X_{TVC} Error (mm/s)	Y_{TVC} Error (mm/s)	1-σ Uncertainty: SMAA $\times$ SMIA, $\theta^\dagger$ (mm/s)
TCM-05	*03-Dec-1998 06:00*	*450.3174*	*-80.57*	*0.38*	*125.60*	*639.93*	*6.12 × 0.12, 89.1°*
TCM-06	04-Feb-1999 20:00	11.5674	-15.80	1.16	11.70	2.61	5.16 × 0.11, 113.6°
TCM-09	*06-Jul-1999 17:00*	*43.5849*	*-85.27*	*9.11*	*35.91*	*77.26*	*4.09 × 0.08, 9.3°*
TCM-10	19-Jul-1999 16:00	5.1443	-5.80	3.83	0.26	10.27	0.48 × 0.09, 43.8°
TCM-11	*02-Aug-1999 21:30*	*36.3446*	*-49.34*	*1.81*	*16.31*	*63.03*	*4.52 × 0.10, 85.7°*
TCM-12	11-Aug-1999 15:30	12.2734	-18.65	3.40	-11.34	16.49	0.76 × 0.14, 83.4°
TCM-13	31-Aug-1999 16:00	6.7231	-17.46	1.85	-7.95	31.82	5.12 × 0.05, 88.1°
TCM-14	14-Jun-2000 17:00	0.5511	-12.52	2.19	0.30	6.53	2.88 × 0.22, 37.7°
TCM-17	28-Feb-2001 17:30	0.5333	-0.72	8.26	-15.16	6.74	10.56 × 6.07, 29.9°
TCM-18	03-Apr-2002 18:00	0.9087	-4.38	3.25	-3.25	1.79	2.89 × 0.46, 92.4°
TCM-19	01-May-2003 20:00	1.6033	1.10	2.88	-8.64	-6.59	4.62 × 3.01, 115.8°
TCM-19b	02-Oct-2003 04:00	2.0012	20.47	4.18	22.20	22.81	5.09 × 0.76, 91.3°
TCM-20	27-May-2004 22:26	34.7543	-43.08	1.88	-38.12	59.44	2.14 × 0.67, 100.3°
TCM-21	16-Jun-2004 21:07	3.7104	-13.54	0.48	3.34	12.53	2.35 × 2.14, 107.1°
OTM-002	*23-Aug-2004 15:53*	*393.1892*	*-134.00*	*2.73*	*454.20*	*-15.10*	5.09 × 4.49, 69.3°
OTM-003	07-Sep-2004 16:30	0.5075	5.85	6.69	-2.05	0.15	6.51 × 1.67, 91.9°
OTM-005	29-Oct-2004 06:15	0.6510	-10.14	0.15	-2.78	0.80	0.87 × 0.11, 91.4°
OTM-006	21-Nov-2004 05:00	0.4158	-0.90	0.59	-2.15	3.65	1.36 × 0.52, 148.9°
OTM-008	17-Dec-2004 01:22	11.9487	-15.23	5.06	2.71	4.75	3.21 × 0.54, 172.8°
OTM-010	28-Dec-2004 00:37	23.7982	-7.09	3.03	-65.93	-20.45	5.52 × 0.51, 91.9°
OTM-011	16-Jan-2005 09:20	21.6466	-4.83	4.18	-16.43	11.68	1.49 × 0.92, 165.1°
OTM-012	28-Jan-2005 07:08	18.7189	-3.92	0.47	-25.16	4.75	1.09 × 0.87, 5.1°
OTM-014	18-Feb-2005 06:00	0.7233	-6.51	0.50	-1.86	5.45	1.77 × 0.03, 90.6°
OTM-015	02-Mar-2005 04:50	6.2603	0.21	2.55	-4.66	6.69	2.00 × 0.14, 106.2°
OTM-017	12-Mar-2005 03:20	0.4555	-2.40	0.73	-0.76	2.87	2.30 × 0.05, 83.1°
OTM-018	19-Mar-2005 18:19	1.6304	-3.78	1.29	-0.49	-1.64	0.91 × 0.30, 0.2°
OTM-020	04-Apr-2005 02:22	0.9267	-8.82	0.85	-0.95	4.83	0.89 × 0.11, 97.0°
OTM-021	10-Apr-2005 02:00	5.8813	-15.67	6.68	12.76	11.01	6.58 × 0.40, 30.0°
OTM-024	29-Apr-2005 00:58	20.5844	5.61	4.73	-8.99	18.84	3.97 × 0.12, 32.0°
OTM-025	08-Jul-2005 20:37	0.3768	-6.35	0.45	-2.99	3.32	6.52 × 0.12, 83.6°
OTM-026	03-Aug-2005 11:50	2.6291	-8.58	2.66	-11.30	0.46	7.73 × 0.04, 87.7°
OTM-027	10-Aug-2005 13:21	2.4176	-3.00	3.10	-7.28	0.68	0.97 × 0.17, 61.1°
OTM-029	25-Aug-2005 17:08	1.4585	-7.28	2.58	-4.27	-2.28	1.86 × 0.22, 102.8°
OTM-030	30-Aug-2005 18:43	14.3573	-2.61	4.25	-7.41	7.20	1.47 × 0.20, 70.6°
OTM-033	*19-Sep-2005 16:40*	*27.9211*	*3.38*	*0.55*	*-59.36*	*6.69*	*3.41 × 0.08, 88.5°*

* Expected $\Delta\mathbf{V}$ includes the design $\Delta\mathbf{V}$ (burn and turns) plus $\Delta\mathbf{V}$ events related to the maneuver.
† 1-σ ellipse dimensions (semi-major axis (SMAA) $\times$ semi-minor axis (SMIA)) with orientation angle, θ (relative to pointing plane X_{TVC} axis).

Table 4 MEA Maneuver Execution Errors (October 2005 - September 2007)

Maneuver *(Excluded)*	Maneuver Epoch (UTC/SCET)	Expected $\Delta\mathbf{V}^*$ (m/s)	Magnitude: Mag. Error (mm/s)	Magnitude: 1-σ Uncert. (mm/s)	Pointing: X_{TVC} Error (mm/s)	Pointing: Y_{TVC} Error (mm/s)	Pointing: 1-σ Uncertainty: SMAA $\times$ SMIA, $\theta^\dagger$ (mm/s)
OTM-038	12-Oct-2005 05:57	14.8428	-5.87	0.09	-15.01	-2.86	0.46 $\times$ 0.06, 79.3°
OTM-041	*31-Oct-2005 13:59*	*12.4247*	*-0.39*	*0.08*	*-22.92*	*3.50*	*1.35 $\times$ 0.03, 87.8°*
OTM-042	13-Nov-2005 14:02	2.1358	-2.63	0.88	1.84	5.89	1.74 $\times$ 0.74, 167.3°
OTM-056	22-Mar-2006 04:19	0.4780	1.03	0.14	-1.13	3.74	0.19 $\times$ 0.09, 45.0°
OTM-057	06-Apr-2006 03:32	0.3654	-3.64	0.05	-2.04	3.93	0.17 $\times$ 0.12, 121.4°
OTM-059	04-May-2006 01:28	0.5096	5.54	0.15	-1.28	2.58	0.17 $\times$ 0.12, 17.2°
OTM-063	07-Jun-2006 23:24	1.9206	-7.86	0.08	-3.56	6.30	0.11 $\times$ 0.04, 83.0°
OTM-069	01-Aug-2006 20:05	5.4243	-3.97	0.70	-8.36	12.63	0.36 $\times$ 0.06, 96.9°
OTM-071	10-Sep-2006 18:00	6.5776	7.57	2.66	-8.21	12.84	4.37 $\times$ 0.20, 131.3°
OTM-072	14-Sep-2006 10:07	8.1704	-5.59	2.72	-3.22	3.47	1.33 $\times$ 0.09, 128.1°
OTM-075	01-Oct-2006 09:08	6.4802	-5.69	0.10	-8.64	2.26	0.09 $\times$ 0.02, 90.7°
OTM-078	17-Oct-2006 15:40	0.8560	-12.73	0.12	-3.88	1.52	0.13 $\times$ 0.08, 100.2°
OTM-080	09-Nov-2006 14:28	3.6767	-6.31	0.03	-0.52	4.91	0.17 $\times$ 0.10, 44.1°
OTM-083	15-Dec-2006 12:03	0.8095	-10.03	0.15	-2.22	0.53	0.37 $\times$ 0.04, 53.7°
OTM-084	20-Dec-2006 11:48	6.8812	-4.12	0.03	-8.27	5.09	0.10 $\times$ 0.07, 143.1°
OTM-086	*31-Dec-2006 11:05*	*0.4996*	*4.63*	*0.10*	*-0.40*	*2.70*	*0.35 $\times$ 0.04, 21.7°*
OTM-087	*05-Jan-2007 10:50*	*1.6644*	*-7.60*	*0.02*	*0.80*	*-2.15*	*0.14 $\times$ 0.10, 57.6°*
OTM-090	21-Jan-2007 09:36	2.3966	2.70	0.05	1.08	3.88	0.14 $\times$ 0.10, 6.8°
OTM-093	07-Feb-2007 08:37	0.2826	-3.77	0.25	-4.90	4.77	0.27 $\times$ 0.02, 88.8°
OTM-096	02-Mar-2007 06:51	0.6569	-0.46	0.07	-1.18	2.29	0.27 $\times$ 0.07, 108.1°
OTM-098	13-Mar-2007 06:06	1.0696	-1.68	0.25	-1.73	1.88	1.29 $\times$ 0.21, 90.1°
OTM-099	18-Mar-2007 05:50	1.6131	-7.50	0.36	-0.75	-1.04	0.83 $\times$ 0.09, 103.3°
OTM-101	28-Mar-2007 20:49	0.5239	9.69	0.29	-1.66	3.72	0.92 $\times$ 0.16, 90.2°
OTM-102	03-Apr-2007 04:34	2.6953	-8.00	0.32	-4.10	2.14	0.51 $\times$ 0.13, 109.1°
OTM-105	19-Apr-2007 03:32	3.5322	-7.81	1.22	-1.10	8.41	1.10 $\times$ 0.58, 155.2°
OTM-108	04-May-2007 19:00	5.5793	-2.99	0.25	2.43	11.04	0.42 $\times$ 0.14, 90.8°
OTM-111	21-May-2007 01:27	5.5285	-0.28	1.05	-8.10	5.13	0.63 $\times$ 0.39, 153.9°
OTM-113	01-Jun-2007 00:41	0.7030	-1.43	0.26	-2.73	0.95	1.18 $\times$ 0.24, 85.0°
OTM-114	05-Jun-2007 16:55	12.2341	-0.53	3.09	-17.16	2.35	1.01 $\times$ 0.59, 160.8°
OTM-116	16-Jun-2007 23:39	0.7607	-8.75	0.97	-1.33	0.98	1.10 $\times$ 0.23, 153.6°
OTM-117	21-Jun-2007 23:23	7.9672	2.13	3.21	-12.81	3.37	0.71 $\times$ 0.38, 44.6°
OTM-123‡	06-Aug-2007 20:35	0.4273	-5.01	0.31	-3.41	0.83	0.30 $\times$ 0.21, 27.4°
OTM-125	02-Sep-2007 11:35	0.4759	2.72	0.46	-4.05	1.30	0.71 $\times$ 0.34, 0.8°
OTM-128	13-Sep-2007 18:20	13.4828	-12.15	2.35	-17.38	2.89	1.10 $\times$ 0.40, 101.6°

* Expected $\Delta\mathbf{V}$ includes the design $\Delta\mathbf{V}$ (burn and turns) plus $\Delta\mathbf{V}$ events related to the maneuver.
† 1-σ ellipse dimensions (semi-major axis (SMAA) $\times$ semi-minor axis (SMIA)) with orientation angle, θ (relative to pointing plane X_{TVC} axis).
‡ Maneuver performed on backup time.

Table 5 RCS Maneuver Execution Errors (September 2003 - September 2007)

Maneuver *(Excluded)*	Maneuver Epoch (UTC/SCET)	Expected $\Delta\mathbf{V}^*$ (m/s)	Magnitude: Mag. Error (mm/s)	Magnitude: 1-σ Uncert. (mm/s)	Pointing: X_{TVC} Error (mm/s)	Pointing: Y_{TVC} Error (mm/s)	Pointing: 1-σ Uncertainty: SMAA $\times$ SMIA, $\theta^\dagger$ (mm/s)
TCM-19a	10-Sep-2003 20:00	0.1251	-2.83	0.14	-2.30	0.99	3.81 × 3.70, 82.8°
OTM-004	23-Oct-2004 06:16	0.3864	0.79	0.63	-4.77	-2.88	1.23 × 0.77, 66.8°
OTM-009	23-Dec-2004 00:52	0.0180	2.68	1.89	-0.21	-0.01	0.50 × 0.36, 89.5°
OTM-010a	03-Jan-2005 23:38	0.1367	2.03	2.23	2.65	0.55	3.32 × 1.20, 51.2°
OTM-013	12-Feb-2005 06:07	0.2088	-1.24	0.83	0.93	0.03	1.94 × 0.45, 73.9°
OTM-022	14-Apr-2005 02:40	0.0652	-0.33	0.06	0.16	1.40	1.02 × 0.11, 96.7°
OTM-031	03-Sep-2005 17:30	0.0628	0.31	0.39	-0.25	-0.10	3.86 × 0.27, 104.6°
OTM-035	28-Sep-2005 16:11	0.3028	-2.96	0.55	3.93	-1.71	0.47 × 0.15, 79.0°
OTM-039	21-Oct-2005 14:58	0.0934	-0.95	0.67	3.32	-0.35	0.75 × 0.07, 97.7°
OTM-043	23-Nov-2005 13:03	0.0631	1.24	0.57	-1.40	0.56	3.16 × 0.17, 85.9°
OTM-044	*28-Nov-2005 04:15*	*0.2412*	*3.13*	*0.10*	*1.51*	*-0.11*	*0.45 × 0.04, 79.1°*
OTM-047	30-Dec-2005 02:47	0.1820	-1.14	0.15	0.67	0.09	0.25 × 0.07, 88.0°
OTM-051	02-Feb-2006 07:53	0.1851	-0.13	0.21	2.17	0.08	0.14 × 0.01, 89.6°
OTM-053	*02-Mar-2006 05:51*	*0.2656*	*-1.73*	*0.12*	*1.35*	*0.16*	*0.14 × 0.02, 90.0°*
OTM-058	27-Apr-2006 01:59	0.0751	2.70	0.10	-0.88	-0.12	0.24 × 0.04, 99.2°
OTM-061	18-May-2006 00:41	0.1138	1.98	0.22	-0.87	0.39	0.41 × 0.20, 81.0°
OTM-064	28-Jun-2006 22:07	0.0675	1.42	0.17	0.80	-0.15	0.15 × 0.01, 93.7°
OTM-065	05-Jul-2006 21:36	0.1372	-1.01	0.08	-0.06	0.24	0.23 × 0.11, 160.8°
OTM-070	04-Sep-2006 18:21	0.2243	-0.56	0.05	0.39	-0.21	0.78 × 0.31, 99.7°
OTM-076	06-Oct-2006 16:24	0.0369	0.36	0.05	-0.02	-0.07	0.14 × 0.12, 5.9°
OTM-079	22-Oct-2006 15:26	0.0501	1.57	0.09	0.33	-0.15	0.21 × 0.18, 39.3°
OTM-081‡	*27-Nov-2006 13:15*	*0.2190*	*-1.30*	*0.05*	*-0.75*	*0.89*	*0.21 × 0.04, 79.3°*
OTM-088	*10-Jan-2007 10:20*	*0.0555*	*-2.28*	*0.04*	*-0.50*	*-0.07*	*0.14 × 0.12, 142.7°*
OTM-089	*16-Jan-2007 02:36*	*0.2192*	*0.98*	*0.09*	*-0.54*	*0.95*	*0.20 × 0.07, 147.0°*
OTM-091	26-Jan-2007 09:21	0.0129	0.05	0.04	0.19	-0.02	0.07 × 0.05, 79.1°
OTM-094	19-Feb-2007 07:37	0.0423	-0.44	0.15	0.77	0.18	0.11 × 0.03, 91.9°
OTM-100	22-Mar-2007 20:30	0.0618	1.01	0.11	0.77	-0.05	0.17 × 0.04, 79.2°
OTM-103	*07-Apr-2007 20:48*	*0.0324*	*1.71*	*0.05*	*1.06*	*0.03*	*0.15 × 0.06, 73.0°*
OTM-106	*24-Apr-2007 03:16*	*0.0123*	*1.15*	*0.17*	*-0.80*	*0.01*	*0.12 × 0.02, 91.5°*
OTM-109	09-May-2007 02:14	0.0268	1.31	0.11	-0.36	-0.03	0.12 × 0.10, 90.8°
OTM-115	11-Jun-2007 00:10	0.0344	-0.78	0.16	-0.04	-0.08	0.12 × 0.07, 161.0°
OTM-118	26-Jun-2007 23:08	0.0134	-0.12	0.02	-0.00	-0.01	0.05 × 0.04, 17.6°
OTM-119	03-Jul-2007 22:37	0.0219	1.64	0.10	0.58	-0.01	0.08 × 0.07, 118.1°
OTM-121	15-Jul-2007 22:06	0.0167	2.00	0.40	-2.06	0.03	0.30 × 0.25, 179.5°
OTM-129	17-Sep-2007 18:21	0.1032	0.03	0.78	0.43	0.06	0.34 × 0.13, 87.4°

* Expected $\Delta\mathbf{V}$ includes the design $\Delta\mathbf{V}$ (burn and turns) plus $\Delta\mathbf{V}$ events related to the maneuver

† 1-σ ellipse dimensions (semi-major axis (SMAA) $\times$ semi-minor axis (SMIA)) with orientation angle, θ (relative to pointing plane X_{TVC} axis).

‡ Maneuver performed on backup time.

OD estimates of the magnitude errors. These uncertainties were used to weight each maneuver in the maximum-likelihood estimator. The red dashed lines in the figures indicate the magnitude-error biases as functions of ΔV magnitude which were computed using the fixed- and proportional-magnitude biases from the study (see table 8). The red solid lines display the 1-σ bounds on the magnitude errors using the 2007-02 study execution-error model given in table 6. All maneuvers considered within the 1-σ magnitude-error bounds are given red error bars. As seen in figure 5, the 1-σ magnitude errors of 18 out of 28 (64%) RCS maneuvers were within the 1-σ magnitude-error bounds ($\approx$68%), which is near the expected normal distribution of magnitude errors as a function of maneuver size. Likewise in figure 3, the 1-σ magnitude errors of 39 out of 61 (64%) MEA maneuvers were within the 1-σ magnitude-error bounds.

The percentage of samples that fall within the estimated standard deviation in each case support the idea that the distributions examined here, both MEA and RCS, are Gaussian on a maneuver-by-maneuver basis. As such, it should be expected that the maximum-likelihood estimator can find a good fit to the data. The trends in the fit seem to be consistent with the data, the resulting model appears reasonable, and the model parameters match well with expectations.

Figures 4 and 6 show the 1-σ pointing error ellipses in the TVC pointing plane of the MEA and RCS maneuvers, respectively, from the new study. The pointing error ellipses are color-coded according to maneuver size (as indicated in the figures).

2007-02 Model and Biases

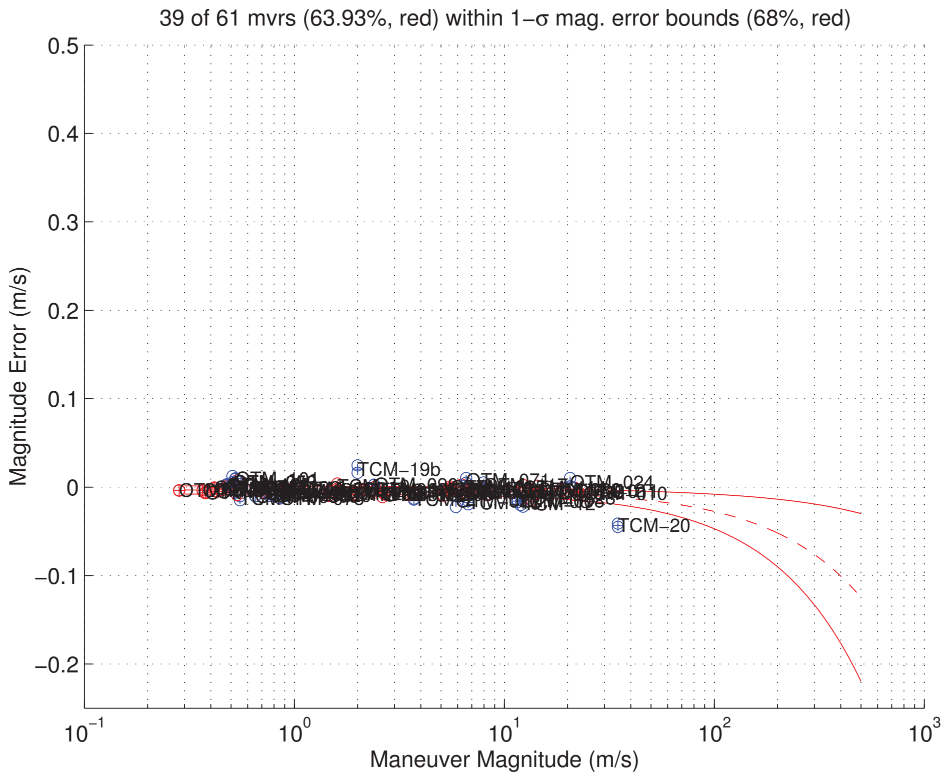
In the 2007-02 execution-error study, data were processed to remove magnitude and pointing biases from the error estimates. Table 8 shows the fixed and proportional components of the magnitude and pointing biases computed for both MEA and RCS maneuvers.

Table 6 2007-02 Maneuver Execution-Error Model (1-σ)

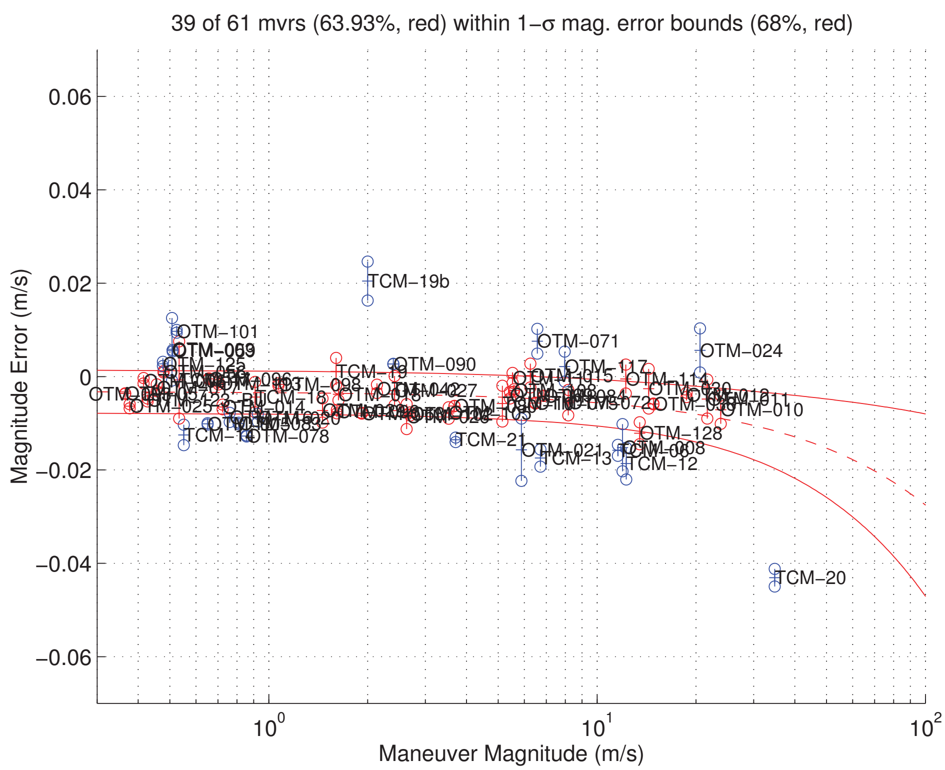
		MEA	RCS
Magnitude	Proportional (%)	0.02	1.2
	Fixed (mm/s)	5.0	0.8
Pointing (per axis)	Proportional (mrad)	0.6	5.5
	Fixed (mm/s)	3.0	0

Table 7 2006-01 Execution-Error Biases

		MEA	RCS
Magnitude	Proportional (%)	0.06	1.5
	Fixed (mm/s)	-4.5	0
Pointing (X_{TVC} axis)	Proportional (mrad)	0.3	7.5
	Fixed (mm/s)	-9.0	0.8
Pointing (Y_{TVC} axis)	Proportional (mrad)	1.5	-4.5
	Fixed (mm/s)	-3.0	3.5

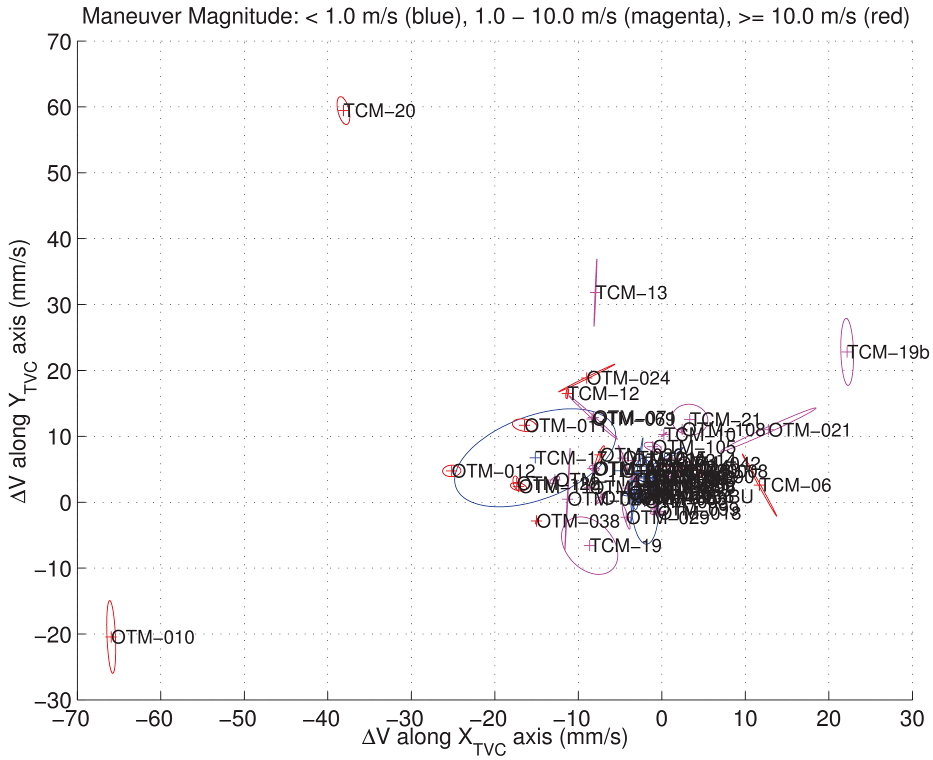


(a) MEA Maneuvers (Cruise through OTM-128)

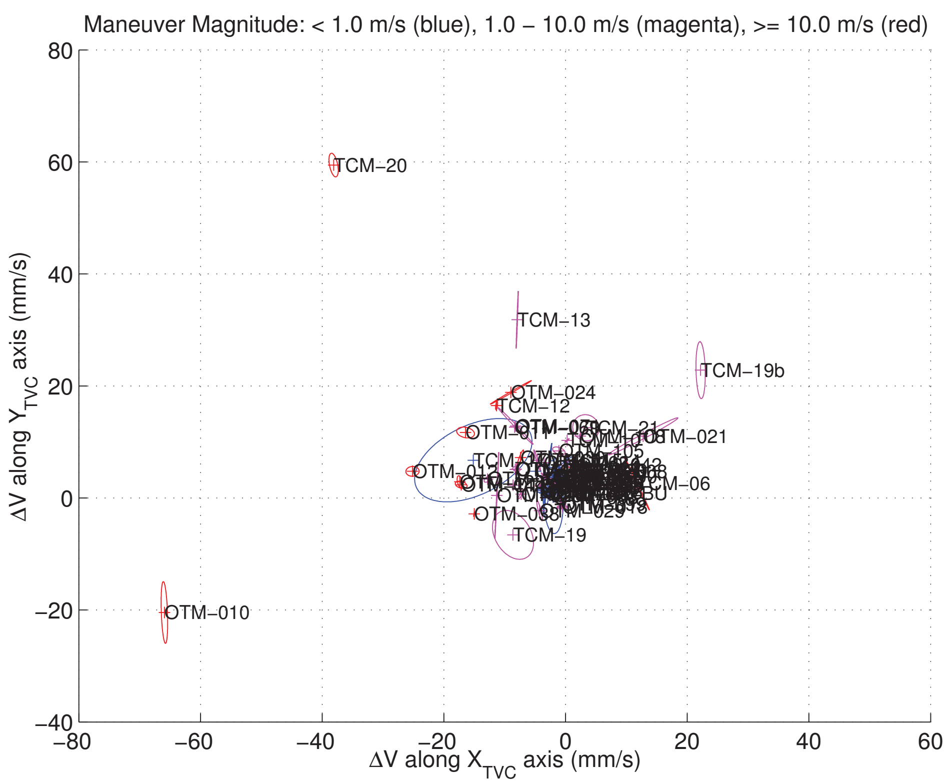


(b) Close-Up of MEA Maneuvers under 100 m/s

Figure 3 MEA Magnitude Errors. *Error bars show 1-σ uncertainties, red dashed lines the magnitude-error biases, and red solid lines the 1-σ magnitude-error bounds. All maneuvers considered within the 1-σ magnitude-error bounds are given red error bars.*



(a) MEA Maneuvers (Cruise through OTM-128)



(b) Close-Up of Dashed Box in (a)

Figure 4 MEA Pointing-Error Ellipses in Pointing Plane (1-σ). *TCM-05 (DSM) and OTM-002 (PRM) are not shown since they are far from the origin.*

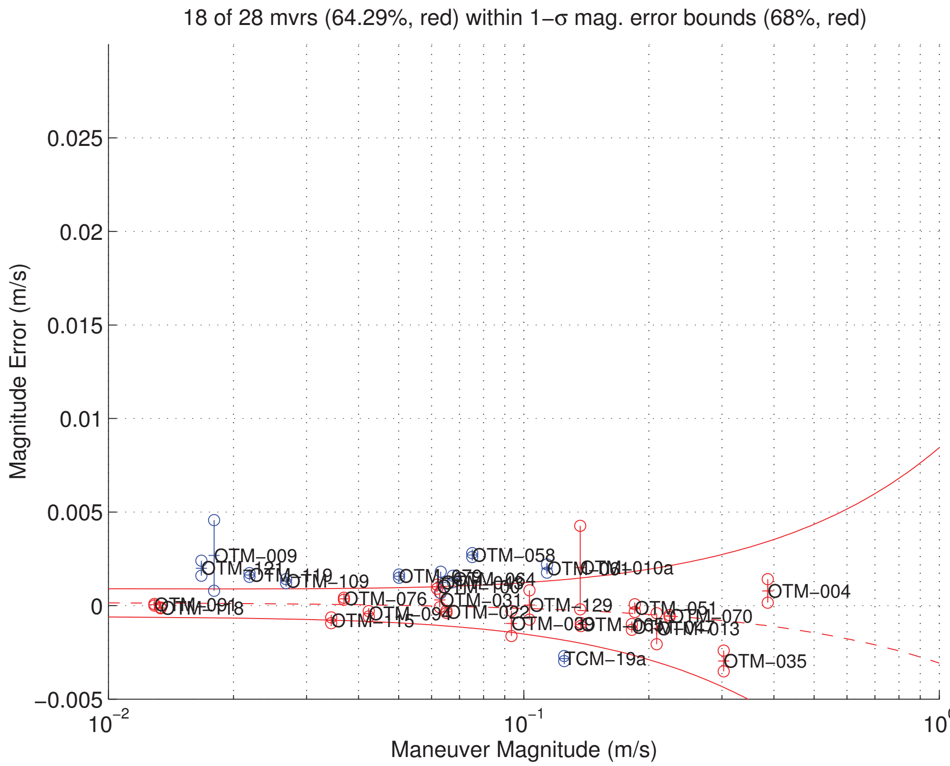


Figure 5 RCS Magnitude Errors (Cruise through OTM-129). *Error bars show the 1-σ uncertainties, red dashed line the magnitude-error bias, and red solid lines the 1-σ magnitude-error bounds. All maneuvers considered within the 1-σ magnitude-error bounds are given red error bars.*

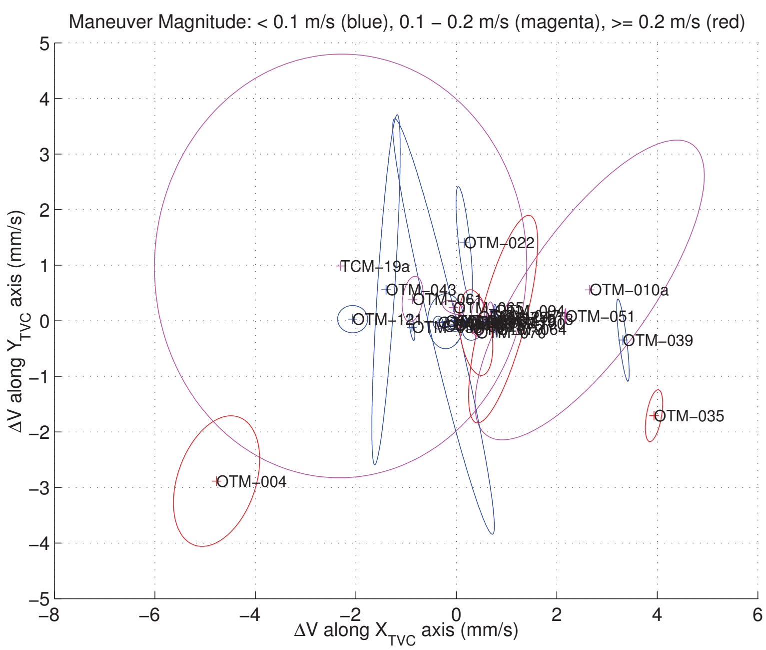
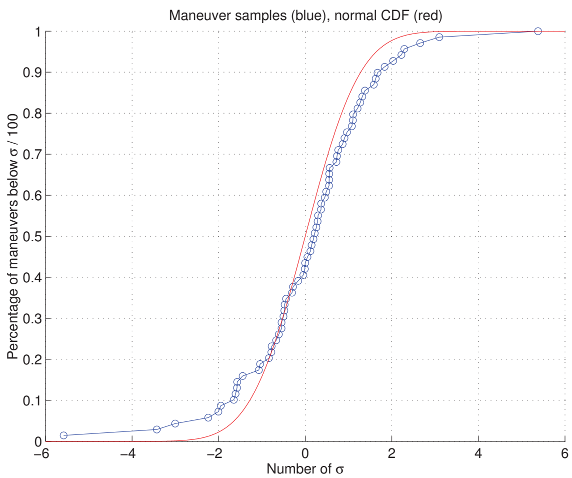
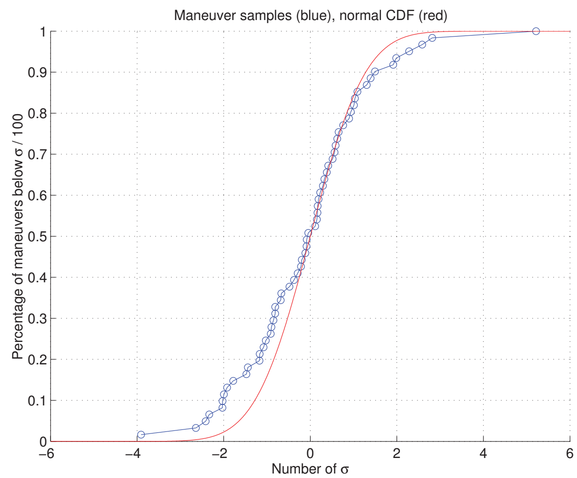


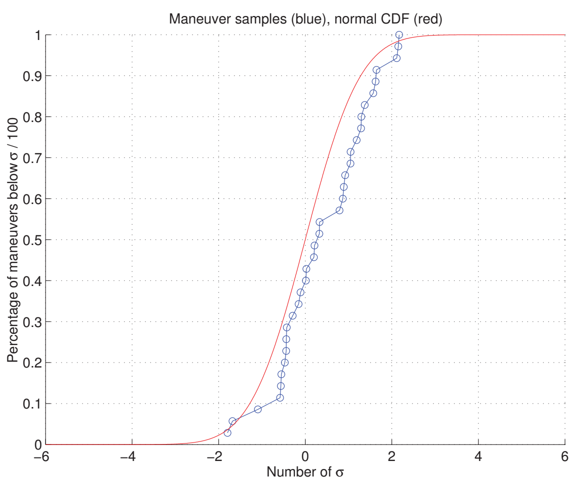
Figure 6 RCS Pointing-Error Ellipses in Pointing Plane (1-σ).



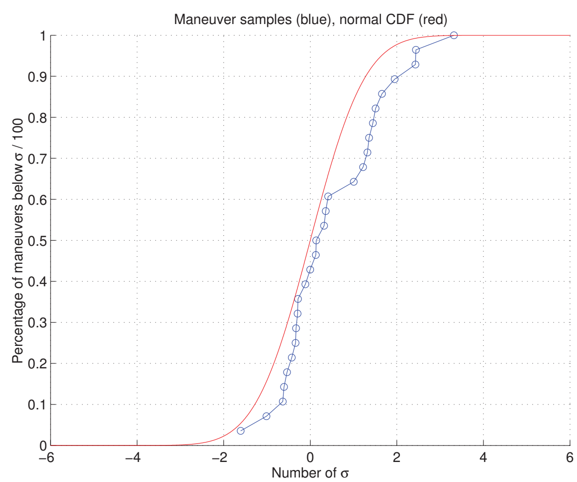
(a) MEA Maneuvers (TCM-01 & SOI Excluded)



(b) MEA Maneuvers (All 2007-02 Exclusions)



(c) RCS Maneuvers (Excluding TCMs 2 & 7)



(d) RCS Maneuvers (All 2007-02 Exclusions)

Figure 7 Magnitude-Error Cumulative Distribution Functions. *(b) and (d) correspond to the results using the 2006-01 study maneuver exclusions. (a) and (c) correspond to the results using the 2007-02 study maneuver exclusions.*

Table 8 2007-02 Execution-Error Biases

		MEA	RCS
Magnitude	Proportional (%)	-0.02	-0.3
	Fixed (mm/s)	-3.5	0
Pointing (X_{TVC} axis)	Proportional (mrad)	-0.7	12.0
	Fixed (mm/s)	-3.5	0
Pointing (Y_{TVC} axis)	Proportional (mrad)	0.7	-1.2
	Fixed (mm/s)	1.2	0

FUTURE WORK

There are several considerations that will be made in future updates of the execution-error model that were not made with this and previous studies for various reasons. The following is a list of known issues to be addressed with future models:

- Investigate why MEA maneuvers tend to underburn.
- Revisit the possibility of separate MEA models for the two post-OTM-111 RCS yaw turn rates.
- Incorporate TCM-02 and TCM-07 in modeling.
- Further investigate the processing of TCM-05 (Deep Space Maneuver, DSM) and OTM-002 (Periapsis Raise Maneuver, PRM).
- Further investigate MEA and RCS outliers maneuvers, drawing from experience gained from the 2007-02 analysis.

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